The inextricable multidisciplinary complexity in human population dynamics modeling

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Sylvie, Birth Day Conference,

IHP, September 11, 2018

Sylvie in June 2004

My first discussion with Sylvie

The context

- ENS Fontenay (1980) where I was Prof since 1979
- In charge of the orientation of students (girls, (up to 1981))
- Moderate opening towards research (from the administration)
- Individual orientation entretien with all students

The meeting

- Sylvie arrives very determined to explain to me that
- her passion is to do sewing and not math's.
- I say: OK but only if you are the best (scientific) in this domain
- Then, go to Dior or Chanel first, work with their new technologies and after, made "fashion sewing" if you want....

Finally, she continued to do math's and sewing, but this last only for the pleasure, and both with great talent

Women in the Laboratoire de Probabilité

Attractive pole

- 1979-1986 : Prof ENS Font (without girls after 1981)
- Research Group in Control and Filtering (1976)
- Sylvie M. : PhD Thesis (1984) on Martingales technics for one dimensional Markov process
- Sylvie R. : PhD Thesis
- Sylvie, (as Sarah now) gets a job at Le Mans

Reverse genealogy, (outside of LPMA)

- Sylvie succeeds in all domains
- The last one is interactions maths-ecology-bio
- She has strongly inspired our modeling of human populations

I am now the "math mother" of Sylvie M., and Sylvie R. and many other. Thank you to you.

Sylvie M. and Sylvie.R at my own BirthDay, IHP 2 June 2004.

Why so many questions

about human populations ?

An historic "demographic transition"

Growth of world population

- The past two centuries ($\sim$ 6 generations) presented major changes in world population, by reduction mortality and fertility
  - From 1 billion in 1800, to more than 7 billion today.
  - Life expectancy at birth has grow of 40 years in 150 years.
  - Fertility declines specially in developed countries

Multiple societal, economic, scientific, international challenges

- Population ageing and uncertainty on longevity risk imply
  - political, public health, pension revolutions (govies, private sector)
  - Impacts on the democracy of generational imbalance, climat...
- Important tradition of data collection
  - (Villermé(1830), GRO ($\sim$ 1850)). (Highlight of past)
  - Now, multiple available database (INED, UN, WHO, HMD, ...).
  - More than 50 reports /year by public and private institutions.
- Recent breaks in the evolution and identified need of new models

Demography

"Science" of the populations ?

An Historical Insight

by Hervé Le Bras, (2013)

The three demographies

Three Crucial Dates in demography

- 1661: Graunt Book, *Natural and Political Observations*
  - the first statistic death table by years and causes in London
  - the first Life tables
- 1825: B.Gompertz (Insurance Cie) Mathematical law for the age related mortality rate, still valuable (biology)
- 1907: A.Lotka (Insurance Cie) Founding the theory of stable human population by introducing first
  - the fecondity rate by age of women which jointed to life tables gives the growth rate of the population.
  - the concept of stable population, where if the growth rate is nul, the population is the product of yearly births with life expectancy.
  - still the actual basis for indirect methods of demographic projection
- 1927: International Labour Office Meeting Creation of International Union Scientific Study of Population (CIUSSP)

The Three components of Demography

Statistical Demography and Mathematical Demography

- Study of demographic parameters, mortality rate, life tables,
- Sensitive to political views
- Weak interplay between the qualitative and mathematics

Political Demography

- Central concept of fertility (Malthus, Eugenisme negatif,..)
- Creation of the International UnionScientific Study of Population
- Instituts , INSEE, INED.

Mixing of the three components

- Developed countries are concerned by population (M.Foucault, 2004)
- Methods for projections in the future, by components
- Big Data without models

Taking into account heterogeneity

Heterogeneity and estimation or forecast of mortality rates:

- High amount of data at the national level → lower variance.
- **But** bigger populations implies higher heterogeneity. Trade-off:
  - Behaviors deviate further from average → increase of variance.
  - Heterogeneity of the population changes through time.

Consequences

- Not taking into account heterogeneity can lead to:
  - Increased inequalities due public health reforms (Arnold Bajekal et al. (2017)) or redistribution properties of state pensions.
  - Bias in computation of regulatory reserves.
- Better understanding of heterogeneity allows for comparison between populations of far different compositions ⇒ Basis risk

A shift in paradigm in human populations...

Diverging trends in longevity documented at multiple levels:

- Countries with similar mortality experience until the 80s now diverge (Gaps in female life expectancy at 50 in 10 high-income countries: $\leqslant 1$ year in 1980, $\geqslant 5$ years in 2007, source: HMD).
- Widening of socioeconomic and geographical mortality inequalities.(Gap in male life expectancy at 65 between higher managerial and routine occupations (England Wales): 2.4 years 1982-1986, 3.9 years 2007-2011, ONS)

Comment of National Research Council (2011):

> *"What is perhaps more surprising is that large differences [...] began relatively abruptly around 1980, and that it has taken so long for this divergence to be recognized and analyzed."*

Two databases/New

- 1981-2007: Department of Applied Health Research, UCL.
- 2001-2015: Office for National Statistics, released April 2017).

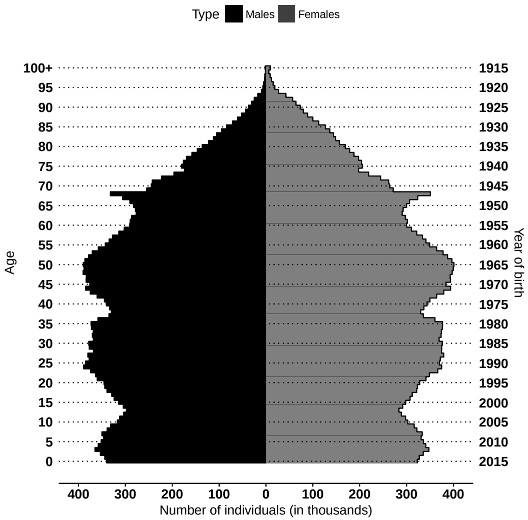
- English cause-specific number of deaths and mid-year population estimates per socioeconomic circumstances, age and gender.

Index of multiple deprivation (IMD)

Socioeconomic circumstances are measured, based on the postcode.

- Small areas (LSOA) are ranked based on seven broad criteria: income, employment, health, education, barriers to housing and services, living environment and crime.
- This ranking permits to divide the population in 5 quintiles with about same number of individuals in each quintile.
- Post-code based index serve as SES proxy and capture information on the environment of individuals.

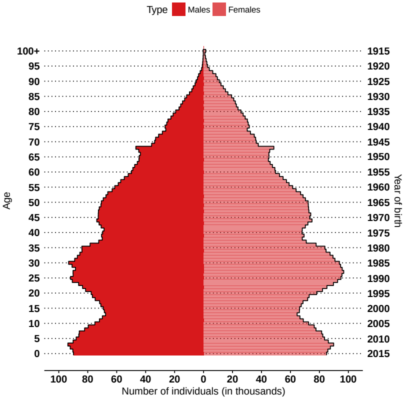
English age pyramid, 2015



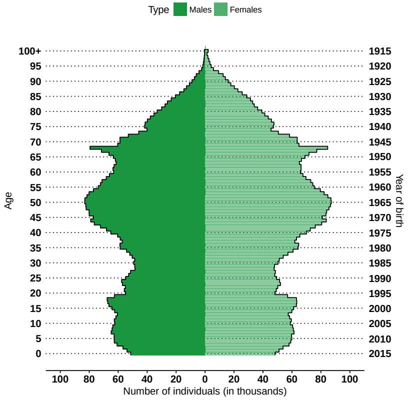
Median age: 39y.

Population composition (− → +), 2015

Age-pyramids by IMD quintile, 2015



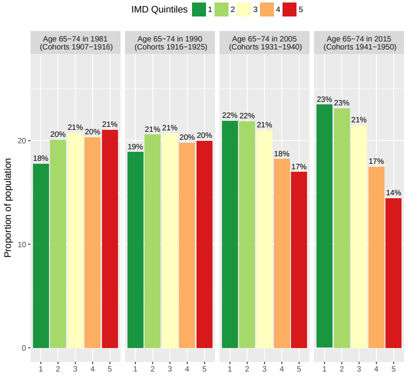
(a) Most deprived quintile ($)
Median age: 33y



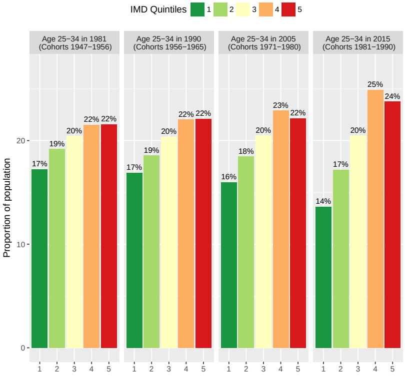
(b) Least deprived quintile ($$$$$)
Median age: 44.2y

- Baby-boom cohort less deprived than younger/older cohorts.

Figure: Composition of males age classes in years **1981, 1990, 2005, 2015**.



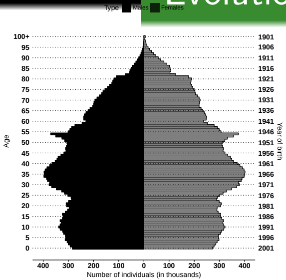
(a) Age class **65-74**



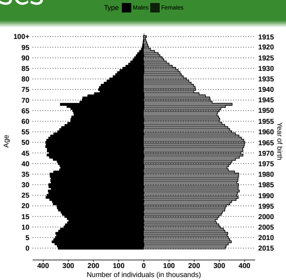
(b) Age class **25-34**

- Decrease of deprivation over time for older age classes, (IMD 1+2: 28% → 46%).

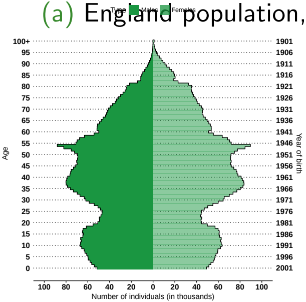
Evolution by classes



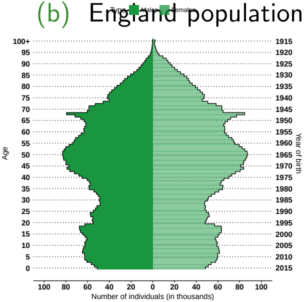
(a) England population, 2001



(b) England population, 2015

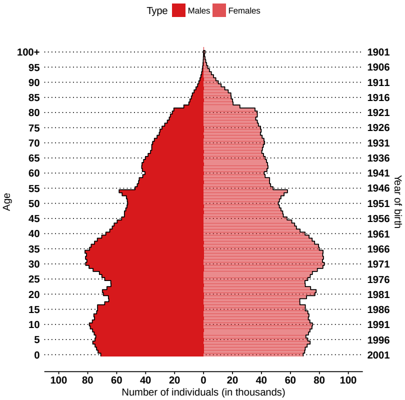


(c) Least deprived quintile ($$$$$)

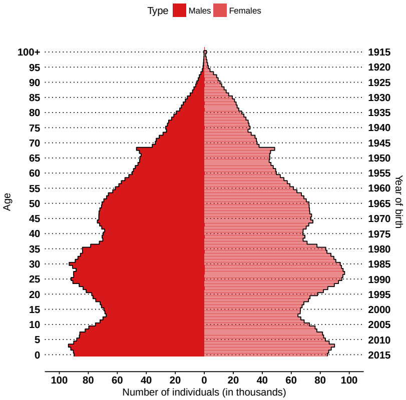


(d) Least deprived quintile ($$$$$)

Most Deprived Quintile



(a) Most deprived quintile ($), 2001



(b) Most deprived quintile ($), 2015

Figure: Age pyramids in 2001 and 2015

Structured population by traits and age

Modelisation strongly inspired by the papers of Sylvie and Tran

Qualitative properties

- Data show strong dependency from the past
- Generational effect
- Cohort and environmental effect

Mathematical Challenges

- to generalize the known methods to non Markovian case (ex stable convergence)
- to develop efficient algorithms
- "Mescoscopic" scale:
  - Identify the level of aggregation: description of subgroups rather than individual life courses.
  - to be carefull to the neighborhood effect (Hierarchical models?)

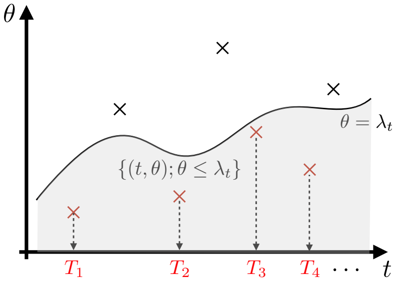
Population viewed as a point process, characterized by traits and age.

- Birth Death Swap system (BDSs) finite numbers of traits is a population process $Z = Z_0 + \phi \odot \mathbf{N}$
  - from a jumps counting system $\mathbf{N} = (N^\gamma)$
  - with non linear class of multivariate intensity $\boldsymbol{\mu}(t, Z_0 + \phi \odot \mathbf{N}_t) = (\mu^\gamma(\omega, t, Z_t))_{\gamma \in \mathcal{J}}$.

Counting System Intensities

- Predictable intensity functional $\boldsymbol{\mu}(t, z)$.
- Support condition (no death or move from an empty class):
$$\mu^{i\beta}(t, z)\mathbf{1}_{\{z^i=0\}} \equiv 0 \quad \forall i \in \mathcal{J}_p,\ \beta \in \mathcal{J}^{(i)}.$$
- Example of linear birth intensity:
$$\mu^{b,i}(\omega, t, z) = b^i_t(\omega)z^i + \underbrace{i_t}_{\text{immigration rate}}.$$

Thinning of Poisson measure



- $N_t^\lambda = \int_0^t \int_{\mathbb{R}+} \mathbf{1}_{]0,\lambda_s]}(\theta) Q(\mathrm{d}s, \mathrm{d}\theta)$ is a *counting process* of $\mathcal{G}_t$-intensity λ_t.
- Marked random measure: $Q^\lambda(\mathrm{d}t, \mathrm{d}\theta) = \mathbf{1}_{]0,\lambda_t]}(\theta) Q(\mathrm{d}t, \mathrm{d}\theta)$
- $\Rightarrow$ Q^λ jump times can be enumerated increasingly (same jump times than N^λ).

BDS stochastic differential system

Idea: control the birth part $\mathbf{N}^b$ of $\mathbf{N}$ $\Rightarrow$ size of the population controlled. Pathwise construction of jumps counting system $\mathbf{N}$ of multivariate intensity $(\boldsymbol{\mu}(\omega, t, Z_{t-}))$:

- **Driving multivariate Poisson measures** family of (p+1)p independent Poisson measures $\mathbf{Q}(ds, d\theta) = (Q^\gamma(ds, d\theta))_{\gamma \in \mathcal{J}}$.
- **The BDS differential system** associated with the intensity functional $\boldsymbol{\mu}(t, z) = (\mu^\gamma(t, z))_{\gamma \in \mathcal{J}}$ is defined for any $\gamma \in \mathcal{J}$ by:

$$dN_t^\gamma = Q^\gamma(dt,]0, \mu^\gamma(t, Z_{t-})]), \quad Z_t = Z_0 + \phi \odot \mathbf{N}_t. \qquad (1)$$

- Vector version:

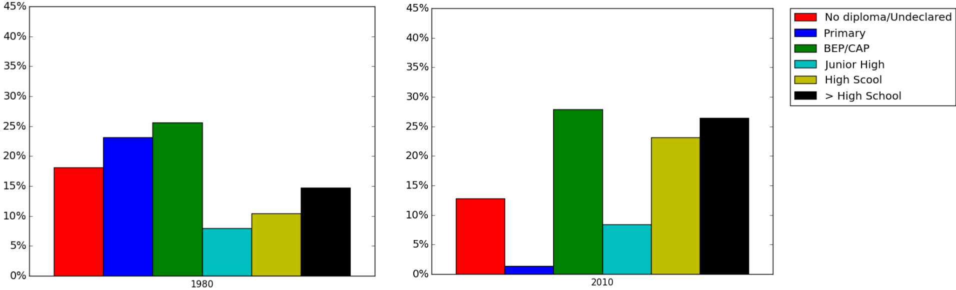
$$d\mathbf{N}_t = \mathbf{Q}(dt,]0, \boldsymbol{\mu}(t, Z_{t-})]), \quad Z_t = Z_0 + \phi \odot \mathbf{N}_t.$$

Mathematics :New Use a Girsanov theorem to analyze the dependency in initial composition of the population

The example of the education level in France

- Age-structured Heterogeneous Population:
  - Individuals are marked by their age and social characteristics (ex: level of education).
  - Patterns of birth rates impact longevity: Cohort effect.
  - Each age and social category has it's own demographic behaviour.
- Why study all ages is interesting in presence of heterogeneity?
  - "Young adults of today are seniors of tomorrow": Social structure of younger age- classes gives lot of information on future structure of seniors population.
  - Generational changes of social structure caused by external factors.
  - Impacts longevity even if specific mortality rates don't change.

Evolution of French Social Structure for 30-45 years old, 1980 → 2010



Data: Echantillon Démographique Permanent, INSEE (National Longitudinal Study)

Modeling Generational Evolution

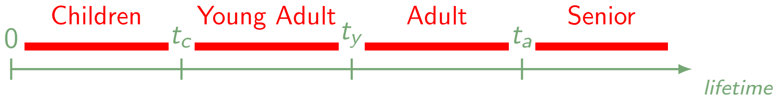
Goal : Modeling and understanding the impact of generational evolution of social structure of the population.

- Example with a population structured in subpopulations and age-classes.
- Generational change occured among Young Adults which will have the possibility of changing of social category.
- Key: difference of time scale between the slow demographic process and fast mixing process.

Usual Age-Structured Population with Heterogeneity

Usual model: M independent subpopulations.

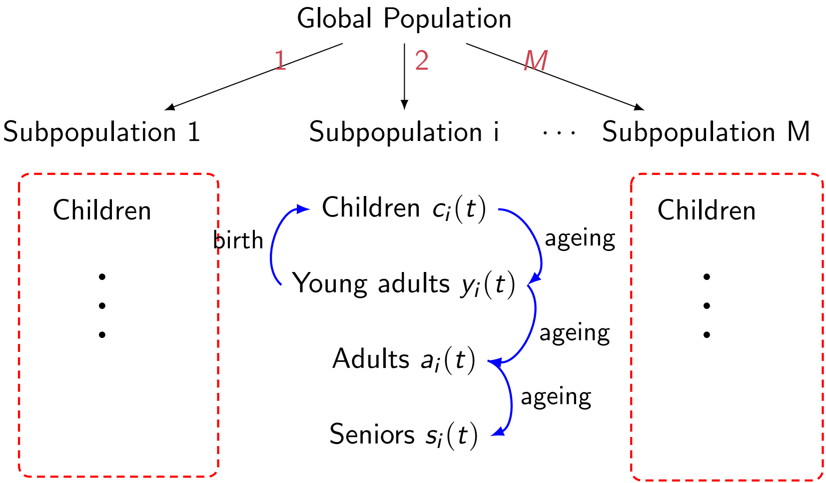
- The population is divided in four ages classes:



- Passive classes = Children, Adults and Seniors:
  - Children inherit the social category of their parents (Ex: Parent with no diploma → child with no diploma).
  - Adults and Seniors don't give birth.
- Active class = Young Adults

  Only young adults give birth.

Demographic Process



Demographic Process

- Number of birth in the category i during $[t, t+dt]$:

$$\beta_i y_i(t)$$

- Number of death during $[t, t+dt]$ (ex for children):

$$d_i^c(t) = \mu_i^c c_i(t)$$

- Ageing during $[t, t+dt]$ (ex for adults, $t_y = 40$ $t_a = 60$):
  - People entering the age-class: $\boxed{S_a^i \beta_i y_i(t-40)}$

    S_a^i = Probability at birth of becoming an adult in class i.
  - People leaving the age-class: $\boxed{S_s^i \beta_i y_i(t-60)}$

    S_s^i = Probability at birth of becoming a senior in class i.

Population Growth Rate

Population Growth Rate in the subpopulation i:

$$
\begin{aligned}
c_i'(t) &= \underbrace{\beta_i y_i(t)}_{\text{birth of young adults' children}} - \underbrace{\mu_i^c c_i(t)}_{\text{children's mortality}} - \underbrace{S_y \beta_i y_i(t - t_c)}_{\text{proportion of children becoming young adults}} \\
y_i'(t) &= S_y^i \beta_i y_i(t - t_c) - \underbrace{\mu_i^y y_i(t)}_{\text{young adults' mortality}} - \underbrace{S_a^i \beta_i y_i(t - t_y)}_{\text{proportion of young adults becoming adults}} \\
a_i'(t) &= S_a^i \beta_i y_i(t - t_y) - \underbrace{\mu_i^a a_i(t)}_{\text{adults' mortality}} - \underbrace{S_s^i \beta_i y_i(t - t_a)}_{\text{proportion of adults becoming seniors}} \\
s_i'(t) &= S_s^i \beta_i y_i(t - t_a) - \underbrace{\mu_i^s s_i(t)}_{\text{seniors' mortality}}
\end{aligned}
$$

Why Young Adults are so important ?

- Dependence of the Seniors' social structure on the past young adult population:

$$s_i'(t) = S_s^i \beta_i \underbrace{\boxed{y_i(t - t_a)}}_{\text{"Parents"}} - \mu_i^s s_i(t)$$

- Death rate of seniors:

$$\mu^s(t) = \sum_{i=1}^M \mu_i^s \boxed{\pi_i^s(t)}$$

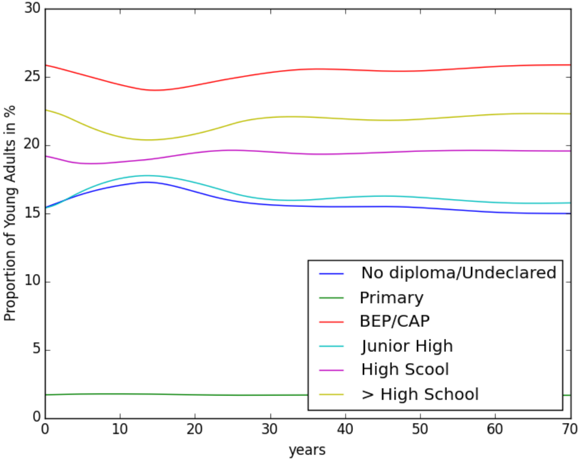
with $\pi_i^s(t) =$ Proportion of seniors in social category i.

- Seniors of Today are Young Adults of Yesterday:

$$\pi_i^s(t) \simeq \pi_i^y(t - (t_a - t_c))$$

Evolution of social Structure of young adults will play important role in future mortality rates

Evolution of young adults $[20, 40]$ social structure with no social mixing



Assumption. There are two time scales in the system for young adults: **slow demographic process and fast mixing process.**

- Example: around 4% of unskilled workers' children obtain a master's degree when the mortality rate for (20-35) years old is of order of $0,05\%$.
- The rate of changing categories is faster than demographic rates for young adults:

$$K \Rightarrow \boxed{\frac{1}{\epsilon}}\, K$$

- Autonomous ODE:

$$\mathbf{y}'_\epsilon(t) = \boxed{\frac{1}{\epsilon}} K \mathbf{y}_\epsilon(t) + \text{Demographic Process}.$$

Interpretation of the Mixing Process

Initial population of young adults: $\mathbf{y}^\epsilon(0) = (y_1^\epsilon(0), .., y_M^\epsilon(0))$. Without demographic process total: number $\bar{y^\epsilon}$ is constant.

- Representative Young Adult: $X^\epsilon(t)$. At $\tau = 0$, $X^\epsilon(t) = i$ with probability $\pi_i^\epsilon(0) = \frac{y_i^\epsilon(0)}{\bar{y^\epsilon}}$. Its "distribution" is $\boldsymbol{\pi}^\epsilon(0) = \frac{\mathbf{y}^\epsilon(0)}{\bar{y^\epsilon}}$.
- Markov Property:

$$\mathrm{P}(X_{t+dt}^\epsilon = j | X_t^\epsilon = i) = \frac{1}{\epsilon} k_{j \leftarrow i} dt + o(dt) \quad \text{if } i \neq j.$$
$$\mathrm{P}(X_{t+dt}^\epsilon = i | X_t^\epsilon = i) = 1 - \frac{1}{\epsilon} k_{ii} dt + o(dt) \quad .$$

Representative individual can be interpreted as a Markov chain in continuous time with intensity matrix ${}^t\frac{1}{\epsilon}\mathbf{K}$. Weak ergodicity: $\lim_{t\to\infty} \boldsymbol{\pi}^\epsilon(t) = \nu$ with ν the stationary distribution of the Markov chain

Stationary Distribution and Approximation

Recall that when there is no demographic process:

- $\boldsymbol{\pi}_\epsilon(t) = \frac{\mathbf{y}_\epsilon}{\bar{y}_\epsilon}$ proportion of young adults in each social category.
- This distribution is characterized by the ODE:
$$\boldsymbol{\pi}'_\epsilon(t) = \tfrac{1}{\epsilon} K \boldsymbol{\pi}_\epsilon(t).$$
- Let $\tau = \frac{t}{\epsilon}$, $\boldsymbol{\pi}_\epsilon(t) = \boldsymbol{\pi}(\tau)$ where $\boldsymbol{\pi}(\tau)$ is the distribution of $X(\tau)$, with X is the representative individual with social mixing K.
- $\lim_{t \to 0} \boldsymbol{\pi}_\epsilon(t) = \lim_{\epsilon \to 0} \boldsymbol{\pi}(\frac{t}{\epsilon}) = \boldsymbol{\nu}$. Thus For ϵ small enough:
$$\boldsymbol{\pi}_\epsilon(t) \simeq \boldsymbol{\nu}$$

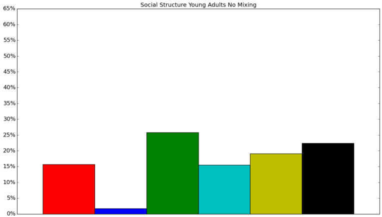
Approximated Model

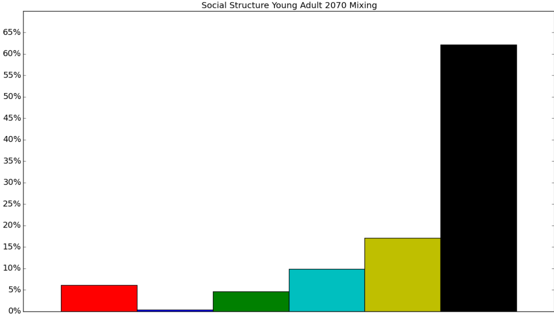
- The result still applies when there is the demographic process: Between 2 demographic events there are enough social mixing so that the social structure stays stable.
- Approximate model: Homegeneous population with mortality/birth rates:

  $$\mu^* = \sum_{i=1}^{M} \boldsymbol{\nu_i} \mu_i \quad \beta^* = \sum_{i=1}^{M} \boldsymbol{\nu_i} \beta_i$$

  where ν is the stationary distribution of the Social Mixing process.
- The stable social structure is not the initial social structure like in the model when there is no mixing.

Comparison of Numerical Results





impact on the mortality rates of 10%.